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LETTER TO THE EDITOR

Massive spin 2 particles from spontaneously broken symmetries

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Abstract. We demonstrate how a spontaneously broken gauge symmetry can be used to produce a massive, gauge invariant, spin 2 theory.

Recent interest in spontaneously broken symmetries has been centred on the Higgs–Kibble phenomenon and the way it generates massive gauge particles in preference to the Goldstone bosons (Higgs 1966, Kibble 1967). A simple example of how this occurs can be seen in nonlinear theories where the renormalizability of the theory is sacrificed, but the phenomenon of the induced mass is obvious, and relies solely on the fact that the nonlinearly transforming fields can be completely gauged away (Gottlieb 1973).

In this paper we wish to demonstrate the similarity, in simple models, between gauge theories of spin 1 and spin 2, and thus how to generate massive spin 2 (Salam 1973 and references therein).

By way of introduction, we study first the spin 1 example in the simplest possible model. The lagrangian

$$\mathscr{L} = -\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi \tag{1}$$

is invariant under the nonlinear transformation $\phi \to \phi+\Lambda$. The nonlinear transformation implies that we are considering a spontaneously broken symmetry. When Λ becomes x dependent the lagrangian is no longer invariant and we introduce a vector field with second type gauge transformation

$$A_\mu \to A_\mu + \frac{1}{m}\partial_\mu\Lambda. \tag{2}$$

The derivative $F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu$ is invariant under this transformation and thus we can write

$$\mathscr{L} = -\tfrac{1}{2}[(\partial^\mu\phi - mA^\mu)(\partial_\mu\phi - mA_\mu) + \tfrac{1}{2}F_{\mu\nu}(A)F^{\mu\nu}(A)] \tag{3}$$

as the gauge invariant lagrangian constructed from the ϕ and A_μ fields. However, this is well known as describing a massive spin 1 particle and can be put into the more familiar form by defining $B_\mu = A_\mu - (1/m)\partial_\mu\phi$ which is a group scalar, and thus gives an invariant

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}(B)F^{\mu\nu}(B) - \tfrac{1}{2}m^2B_\mu B^\mu. \tag{4}$$

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So far the mechanism is well known. In order to see the same occurrence in the spin 2 case, we consider the simple massless spin 1 lagrangian

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A) \tag{5}$$

invariant under the transformation $A_\mu \to A_\mu + \xi_\mu$ and thus mimicking the first example. Using the freedom to alter the lagrangian by a total divergence we add

$$-\partial^\nu(A^\mu\partial_\mu A_\nu) + \partial^\nu(A_\nu\partial_\mu A^\mu) \tag{6}$$

so that

$$\mathscr{L} = [-\tfrac{1}{4}(\partial^\mu A^\nu + \partial^\nu A^\mu)(\partial_\mu A_\nu + \partial_\nu A_\mu) + (\partial_\mu A^\mu)^2] \tag{7}$$

and we are now in a position to see exactly how spin 2 plays the same role with this lagrangian as spin 1 has in the simpler model. When the gauge becomes x dependent, the lagrangian is no longer invariant and we can introduce a symmetric tensor field $\phi_{\mu\nu}$ such that

$$\phi_{\mu\nu} \to \phi_{\mu\nu} + \frac{1}{\kappa}(\partial_\mu\xi_\nu + \partial_\nu\xi_\mu) \tag{8}$$

then $\partial_\mu A_\nu + \partial_\nu A_\mu - \kappa\phi_{\mu\nu}$ is a scalar under this transformation. Thus we find that the lagrangian formed by adding a massless spin 2 lagrangian to the invariants constructed from the covariant derivatives of A_μ is

$$\begin{aligned}\mathscr{L} = {} & \tfrac{1}{2}(g^{\alpha\beta\gamma}g_{\beta\gamma\alpha} - {}^{\alpha}g g_\alpha) + (\partial_\mu A^\mu - \tfrac{1}{2}\kappa\phi_\mu{}^\mu)^2 \\ & -\tfrac{1}{4}(\partial^\mu A^\nu + \partial^\nu A^\mu - \kappa\phi^{\mu\nu})(\partial_\mu A_\nu + \partial_\nu A_\mu - \kappa\phi_{\mu\nu})\end{aligned} \tag{9}$$

where $g^{\alpha\beta\gamma} = \partial^\alpha\phi^{\beta\gamma} + \partial^\beta\phi^{\alpha\gamma} - \partial^\gamma\phi^{\alpha\beta}$, $g_\alpha = g_{\alpha\beta}{}^{\beta}$, and ${}_\alpha g = g^\beta{}_{\beta\alpha}$, and is invariant up to a total divergence which thus leaves the action unchanged and can be neglected. The similarity between this lagrangian and the one in equation (3) is obvious. Introducing the field

$$\Phi_{\mu\nu} = \phi_{\mu\nu} - \frac{1}{\kappa}(\partial_\mu A_\nu + \partial_\nu A_\mu) \tag{10}$$

which is a group scalar, we can write equation (9) as

$$\mathscr{L} = \tfrac{1}{2}(G^{\alpha\beta\gamma}G_{\beta\gamma\alpha} - {}^{\alpha}G G_\alpha) - \tfrac{1}{4}\kappa^2\Phi_{\mu\nu}\Phi^{\mu\nu} + \tfrac{1}{4}\kappa^2(\Phi_\mu{}^\mu)^2 \tag{11}$$

where $G^{\alpha\beta\gamma} \equiv g^{\alpha\beta\gamma}(\phi \to \Phi)$, which is just the lagrangian for a spin 2 field with mass $\kappa/\sqrt{2}$.

It is hoped that this simple example demonstrates how the spin 2 field can be given a mass by means of the Higgs mechanism, and provides some further insight into this phenomenon.

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